Greek Arithmology: Pythagoras or Plato?

Leonid Zhmud – Russian Academy of Sciences, Saint Petersburg

1 The problem

There is a genre of ancient Greek and Latin writings which are usually called arithmological. Their best known example is the *Theology of Arithmetic*, a short anonymous treatise of the fourth century AD dealing with the wonderful properties of the first ten numbers. This treatise, mistakenly attributed to Iamblichus, heavily relies on two earlier works, an extant *On the Decad* by Iamblichus' teacher Anatolius and a lost *Theology of Arithmetic* by the Neopythagorean Nicomachus of Gerasa. Several quotations from this work will suffice to give an idea of the overall character of the genre:

> The Pythagoreans called the monad 'intellect' (*nous*) because they thought that intellect is akin to the one; for among the virtues, they likened the monad to moral wisdom; for what is correct is one…
>
> The dyad is also an element in the composition of all things, an element which is opposed to the monad, and for this reason the dyad is perpetually subordinated to the monad, as matter to form…
>
> The triad, the first odd number, is called perfect by some, because it is the first number to signify the totality – the beginning, middle, and end. When people exalt extraordinary events, they derive words from the triad and talk of 'thrice blessed', 'thrice fortunate'. Prayers and libations are performed three times. Triangles both reflect and are the first substantiation of being plane; and there are three kinds of triangle – equilateral, isosceles, and scalene (Waterfield 1988, 39, 42, 51).

Such and similar comments on the philosophical, theological, and mathematical properties of the first ten numbers constitute the bulk of the arithmological works or passages.

Though the term "arithmology" is widely present in the scholarly literature, its meaning sometimes tends to be rather fluid. To avoid misunderstandings I would like to remind readers of the original and still normative sense of the term. It was coined by A. Delatte (1915, 139), who, in his book on Pythagorean literature, defined arithmology as "a genre of notes on the formation, significance, and importance of the first ten numbers, in which sound scientific research is mingled with fantasies of religion and of philosophy." Thus, from the very beginning the term "arithmology" was attached to a *specific genre of non-mathematical writings on the first ten numbers*. It is in this meaning that arithmology was normally used after Delatte, for example by F. Robbins,[1] who in the 1920s investigated most Greek and Latin arithmological texts, spanning from the time of Varro (116–128 BC) to the early Byzantine writers, and established their common ancestor, a pseudo-Pythagorean treatise, probably of the late second or early first century BC. Robbins' results were generally accepted, giving wide currency to the term "arithmology;" whereas his and Delatte's contention that this genre goes back to Pythagoras and the ancient Pythagoreans, only strengthened an already dominant opinion on this question. To be sure, Delatte admitted that the first specimen of the genre was a short treatise *On Pythagorean Numbers* by Plato's nephew and successor Speusippus, and Robbins substantiated his claim by referring to the fragments and book titles of Philolaus and Archytas, which are now universally considered to be spurious.[2] K. Staehle (1931, 3–5), in his useful study of Philo's arithmology and the later parallels to it, was inclined rather to regard the Early Academy as the *Sitz im Leben* of arithmology, but did not develop this idea. It is therefore the purpose of this paper to argue that:

1) the late Hellenistic pseudo-Pythagorean treatise (hereafter *Anonymus Arithmologicus*, *An. Ar.*) offered Platonism disguised as authentic Pythagoreanism, thus sharing a common feature with most pseudo- and Neopythagorean writings of the first century BC – first century AD;[3]
2) arithmology as a system was created in the Early Academy; the principal impetus for its formation came from Plato, especially from his unwritten doctrine of the ten ideal numbers;
3) the interest of the Pythagoreans in significant numbers belonged to traditional Greek number symbolism; even if it influenced Plato and his students, which is not certain, it was different in kind from arithmology.

2 What is arithmology?

When writing about arithmology, it is convenient to start with some general remarks on its nature. As a literary genre arithmology is easily distinguishable from a much more general cultural phenomenon usually called number symbolism (the *Zahlensymbolik* of early nineteenth-century German philosophy), which in ancient Greece gave rise to many diverse practices, such as medical prognostics based on odd and even numbers, embryological calendars, isopsephy, and so on.[4] Some scholars, however, do not see much difference between arithmol-

1 Robbins 1920, 309 n. 1; Robbins 1921. Staehle 1931, 1–2, also endorses Delatte's definition.
2 Delatte 1915, 140; Robbins 1926, 90.
3 See on this my paper "What is Pythagorean in the pseudo-Pythagorean literature?" (in preparation).
4 Another term for this phenomenon, "numerology" (coined in 1907), covers an even vaster area, dealing with the mystical properties of numbers, which includes many modern para-

ogy and number symbolism and use the terms interchangeably;[5] others define them, respectively, as "the body of lore" and as "the method whereby such lore is used,"[6] which is not particularly helpful. What is important, of course, is not the terminology as such, but the need to draw a distinction between a very general psychological habit and its specific literary embodiment. Number symbolism is rooted in human nature[7] and therefore universally widespread. It goes back to preliterate times,[8] whereas arithmology appears in ancient Greece in a specific period and milieu, so that every ancient arithmological text or passage displays manifest affinities with its distant forefather. Thus, all fully preserved Greek arithmological texts, either long or short, start at one and go up to ten; arithmological fragments presuppose the same structure. The only prominent exception is Philo of Alexandria: in his lost work *On Numbers* he commented on practically every number mentioned in Jewish Scripture; still, most of his speculations are confined to the first ten numbers (Staehle 1931).

Traditional number symbolism, be it ancient Near Eastern or Greek, concentrates on *individual significant* numbers, such as three,[9] seven, or nine,[10] which acquired their special significance before and apart from any philosophy. In the framework of number symbolism, numbers are not yet related to the decad, they possess their own independent meaning, whereas arithmology organizes them into the system of the first ten numbers, and treats both their purely mathematical properties and their philosophical and theological implications. In arithmology every number becomes a member of the arithmetical progression from one to ten: one is the beginning of numbers, two is the first even number, three is the first odd number, four is the first square number, and ten is the perfect number, comprising the whole nature of numbers. Whereas number symbolism focuses on various correspondences of the numbers with the things of the outer world (three Moirai, four seasons, seven stars of the Bear, nine Muses), arithmology, retaining this focus, also displays a keen interest in those properties of numbers that are easily amenable to paramathematical interpretations: odd, even, prime, composite, and so on. Two is the first female number and three is the first male number, for even and odd is associated with female and male, five is marriage, seven is the Maiden Athena, for inside the decad it neither produces nor is produced, and so on. Thus, numbers constitute in arithmology an independent level

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mathematical fancies such as pyramidology, etc. See a critical study by an eminent mathematician: Dudley 1997.

5 As, e. g., Burkert 1972, 466 n. 2; Kalvesmaki 2013, 5.

6 Runia 2001, 26–27.

7 In a famous paper the cognitive psychologist George A. Miller explained the ubiquity of seven by the capacity of human memory (Miller 1956). See also Zvi 1988.

8 Rich material was collected by Burkert 1972, 466–474.

9 The fundamental monograph on the triad is Usener 1903. See also Lease 1919, who brings an impressive number of examples among which is "even grammar with its 3 persons, 3 numbers, 3 voices, 3 genders, 3 degrees of comparison, 3 kinds of accent, etc." (67); Mehrlein 1959.

10 Ancient Near East: Dawson 1927; Reinhold 2008. Greece: Roscher 1904; Roscher 1906.

of reality, which demonstrates that this genre could not have originated until Plato developed his theory of the two worlds, the visible and the intelligible, the physical things and the Forms, and until his heirs Speusippus and Xenocrates replaced or identified the Forms and ideal numbers with mathematical numbers.

Understandably, arithmology does not dismiss the traditional meaning of individual significant numbers but incorporates it into its own system. Thus the description of the triad quoted above from *Theology of Arithmetic* (above, 321) goes back to Aristotle's account of the Pythagorean belief in the triad as the number of an "all" (*Ph*. 268a10–20), a belief that certainly derives from the prehistoric lore. Solon's famous elegy on the seven-year ages of man's life (fr. 17 Diehl) was often quoted or alluded to in the arithmological writings. The so-called embryological calendars, which is to say calculations of the development of the foetus, based on the same number as in Solon's scheme, were known in Greek medicine and philosophy from the fifth century BC (see below, 339). Later this practice was partly incorporated into arithmological literature,[11] and partly developed by medical writers.[12] With time, arithmology accumulated many of these traditional beliefs, but what is important to bear in mind is that it has never ousted number symbolism from its traditional niche. Number symbolism continued to live its own life and produce literary specimens of its own kind, such as, for example the late pseudo-Hippocratic treatise *De hebdomadibus* (Roscher 1913). The first part of that tract (ch. 1–11) pays particular tribute to seven without, however, making it a member of a numerical series leading to ten, or even mentioning any other number. Such an approach is typical of number symbolism, not of arithmology.[13] Contrary to *De hebdomadibus*, Varro, who participated in the Neopythagorean movement and definitely used *An. Ar.*,[14] says in an introduction to his *Hebdomades* that, if one adds numbers from 1 to 7 they will make 28, which number is equal to the lunar cycle, that is, to four weeks in seven days.[15] This is what one should normally expect of an arithmological text.

The last general remark on arithmology concerns its similarity to the doxographical genre. Both genres owe their birth to the treatises of the fourth century BC: doxography to the *Opinions of the Natural Philosophers* by Theophrastus, who heavily relied on Aristotle, and arithmology to *On Pythagorean Numbers* by Speusippus, who was no less heavily dependent on Plato. The history of both genres

11 Barker (in this volume).

12 For the general history of this practice, see Parker 1999.

13 The Hellenistic date of *De hebdomadibus*, defended in the thorough study by Mansfeld 1971, remains the most plausible; since connections with first-century BC arithmology have remained unproven, the tract could well have been written in the second century BC. Runia 2001, 280, on different grounds, also suggests the second century BC.

14 Palmer 1970, 19–21. He wrote *On the Principles of Numbers* and *Atticus de numeris* (Cens. *De die nat*. 2.2).

15 Aul. Gel. 3.10.6; cf. 10.13. In Anatolius the same statement sounds like a formula: "When added up from the monad the 7 produces the 28, a perfect number which is equal to its own parts" (11.12–13).

in the third–second centuries is completely unknown, but in the first century BC, the time of the great philosophical revival and change, they re-emerge: one as the anonymous doxographical compendium called by H. Diels the *Vetusta placita*, and another as the *Anonymus Arithmologicus*. Both works represent the decisive turn to philosophy of the classical age; both give rise to a vast family of similar writings. In the same way as doxography, arithmology consists not only of complete writings but more often of passages or parts of texts, the characteristic features of which allow us to identify them in writings of other genres, be it commentaries, philosophical treatises, popular introductions, and so on. The crucial difference between the two genres is apparent: doxography constitutes our most important source for the Presocratics and is a subject of ongoing research and vivid debates, whereas arithmology exists on the margins of the study of Greek philosophy. "The history of arithmology still remains to be written," as it was in 1971, when J. Mansfeld wrote these words.[16] It is far from my intention to write such a history; I shall only attempt to clarify some important issues pertaining to arithmology and its history, starting from *An. Ar.* and going back to the Pythagorean predecessors of Speusippus.

3 The pseudo-Pythagorean writings of the third–second centuries BC

An. Ar., reconstructed in its main features by Robbins, belongs to the pseudo-Pythagorean apocrypha. To what degree does this guarantee that its teaching derives from pre-Platonic Pythagoreanism? The problem concerns not only *An. Ar.* but pseudo-Pythagorica in general, and has given rise to two opposing theories: one sees continuity between ancient and Hellenistic Pythagoreanism and another insists on a rupture between them.[17] After a long discussion it is now widely agreed that pseudo-Pythagorean treatises were fabricated throughout the Hellenistic period and the early Roman empire without any discernible link to the original writings of the Pythagoreans of the fifth and fourth centuries, not to mention Pythagoras' own teaching. When reading pseudo-Pythagorean writers, one gets the impression that they neither knew the works of their proclaimed predecessors, nor were interested in them. That is why the corpus of pseudo-Pythagorica is almost completely useless for any historical reconstruction of the teachings of the ancient Pythagoreans. Revealingly, it contains not a single authentic quotation of Alcmaeon, Philolaus, Archytas, Ecphantus, or any other ancient Pythagorean. Therefore, if a Hellenistic treatise claims to be written by Pythagoras or by any of his ancient followers, we should hardly expect to find in it an authentic Pythagorean teaching. From the turn of the first century BC, pseudo-Pythagorean apocrypha (especially those written in Doric) increasingly relied on Academic and Peripatetic interpretations of Pythagoreanism, or directly on the theories of Plato and Aristotle. This is exactly what we find in *An. Ar.* The tendencies of the

16 Mansfeld 1971, 156. Cf. Runia 2001, 28.
17 For a bibliography of the problem see Zhmud 2012, 6 n. 11; Centrone 2014.

two previous centuries, however, were very different. Though the chronology of the pseudo-Pythagorean writings is notoriously difficult and controversial, a substantial group of them, attributed to Pythagoras himself, are referred to by authors of the third–second centuries and thus can be more or less reliably dated.

The first pseudo-Pythagorean apocrypha started to appear at the end of the fourth century, by which time the school itself had disappeared (after 350). Neanthes of Cyzicus, a historian of the late fourth century BC, mentions the letter to Philolaus, written by the alleged son of Pythagoras, Telauges (*FGrHist* 84 F 26). Neanthes himself considered the letter to be spurious; it seems to have been of biographical, not doctrinal character. The biographer Satyrus (late third century BC) tells the story that Plato bought from Philolaus "three Pythagorean books" published by him, containing the previously unavailable teaching of Pythagoras. This famous *tripartitum* in Ionic prose included the following books: Παιδευτικόν, Πολιτικόν, Φυσικόν (D. L. 8.6, 9, 15); Diogenes Laertius quotes the opening words of the Φυσικόν: "Nay, I swear by the air I breathe, I swear by the water I drink, I will never suffer censure on account of this work." Sotion of Alexandria (ca. 200 BC) in his *Successions of Philosophers* adds to the list of Pythagoras' works two poems, *On the Universe* (Περὶ τοῦ ὅλου) and *Hieros Logos*, as well as *On the Soul*, *On Piety*, *Helothales, the Father of Epicharmus of Cos*, and *Croton* (D. L. 8.7). It is tempting to connect *On the Universe* with an astronomical poem which, according to Callimachus, was falsely ascribed to Pythagoras (Burkert 1972, 307). Cato (*De agric.* 157) and Pliny (*NH* 24.158) relied on a forgery known as *Pythagoras on the Effects of Plants*; Thesleff (1965, 174–177) prints several passages related to this book. One more pseudepigraphon, entitled Κοπίδεζ (D. L. 8.8–10), also belongs to the corpus of diverse writings fabricated under Pythagoras' name before the first century BC. Now, what is available from them is mostly the titles, while information about their content is very meagre. Nevertheless, no scrap of this information is related to arithmology and only one to number symbolism.[18] Similarly to other philosophers of the period, the Hellenistic Pythagoras is said to have written on physics, ethics, politics, and religion, and not on the Monad and Indefinite Dyad. He was at this point in his history still uncontaminated by the early Academic and Aristotelian interpretations of Pythagoreanism, which in the Hellenistic period were either unavailable or were for a long time forgotten as irrelevant.

18 According to the *tripartitum*, the life of a man is divided into four parts of twenty years – a child, an adolescent, a youth, and an adult – which corresponds to the four seasons (D. L. 8.8–10). An analogous passage is to be found in an anonymous biography of Pythagoras in Diodorus Siculus. It is based chiefly on Aristoxenus (Zhmud 2012, 72), in whose *Pythagorean Precepts* the various obligations of the same four age groups were discussed (fr. 35). The *tripartitum* and the *Anonymus Diodori* used the same source. – According to the same *tripartitum*, after 207 years in Hades Pythagoras has returned to the land of the living (D. L. 8, 14). What does this number mean is unclear. Cf. Rohde 1925, 599–600; Thesleff 1965, 171.21.

Attempts to find traces of Pythagorean arithmology in the Jewish historian Aristobulus (mid-second century BC)[19] have been unsuccessful. Aristobulus wrote in the framework of the traditional number symbolism related to the number seven,[20] quoting many Greek poets from Homer to Solon on this account, but characteristically not Pythagoras (or the Pythagoreans), although he maintained that the Greek sage took his philosophy from the Jews (fr. 3a, 4a Holladay). The alleged Pythagorean connection rests solely on a late quotation in Philolaus' spurious work where, similarly to Aristobulus (fr. 5), seven is linked with light.[21] A strained "Pythagorean" interpretation of a verse quoted by Aristobulus from "Linus," ἑβδόμη ἐν πρώτοισι καὶ ἑβδόμη ἐστὶ τελείη (fr. 5) does not add plausibility to this hypothesis either.[22]

The role of Posidonius (ca. 135 – ca. 50 BC) in the emergence and transmission of arithmology was grossly overestimated by A. Schmekel (1892, 409-439), who relied on the fact that two of his fragments are preserved in the context of arithmological speculations by Theon of Smyrna and Sextus Empiricus.[23] By the 1920s Schmekel's critics had convincingly shown that Posidonius did not write an arithmological treatise.[24] However it took much longer to conclude on the basis of his safely attested fragments that Posidonius most probably did not even know the tradition represented by *An. Ar.*[25] His comment on the seven parts of the world soul in Plato's *Timaeus* (fr. 291 E-K) focuses on the correspondences between the number seven and natural events, whereas mathematical and mystical properties of seven or any other number are not mentioned.[26] He praises Plato for following *nature* and, basically, does not add anything new to what is said in the *Timaeus* (35b–36b). Posidonius' comment obviously belongs to the same line of thought that regarded the number seven as φυσικώτατος; it was represented by Solon in the sixth century, Alcmaeon, Heraclitus, Empedocles, Hippon, and the Hippocratic doctors in the fifth, and Plato, Aristotle, and Theophrastus in the fourth (see below, 338). This conclusion has an important chronological corollary:

19 Walter 1964, 155–158; Collins 1984, 1250–1253; Holladay 1995, 224–226.

20 "All the cosmos of all living beings and growing things revolves in series of sevens" (fr. 5).

21 44 A 12, rejected by Burkert 1972, 247 and Huffman 1993, 357. Walter 1964, 155 n. 2, gave credence both to A 12 and to the even more spurious B 20.

22 Holladay 1995, 193, 239 n. 166 follows an erroneous translation: "Seventh is among the prime numbers, and seventh is perfect." 1) Though seven is a prime number (πρῶτος ἀριθμός), ἐν πρώτοις never means "among the prime numbers," but only "in the first numbers that make up the ratios" (2:1, 3:2, etc.), which Euclid defines as "numbers prime to each other" (πρῶτοι πρὸς ἀλλήλους ἀριθμοί, 7, def. 13; Holladay confuses them with the prime numbers). See Eud. fr. 142, Archytas A 16, and Zhmud 2006, 215–218. To be ἐν πρώτοις one needs two numbers, not one. 2) Neither in the Pythagorean nor in the early Academic tradition does seven figure as the perfect number.

23 Fr. 85 E–K = Sext. Emp. 7.93; fr. 291 = Theon. *Intr.* 103.16–104.1.

24 See, e. g. Robbins 1920, 309–320; Staehle 1931, 13–15.

25 For attempts to revive the thesis of Posidonius as the transmitter of arithmology, see Burkert 1972, 54–56; Mansfeld 1971, 156–204. Cf. above, 324.

26 See Edelstein / Kidd 1972–1988, commentaries on fr. 85 and 291.

if Posidonius did not use *An. Ar.*, as Robbins believed, there are no other grounds to date it to the late second century. In fact, its first traces appear in the mid-first century BC.

Hence, no articulated arithmological passage is to be found in the early Hellenistic pseudo-Pythagorean literature or in other authors of this period for whom a connection with the arithmological tradition has been proposed. This is quite remarkable, since the next century brought a veritable flow of such texts. Varro was only twenty years younger than Posidonius, but lived long enough to borrow both from arithmology and from doxography, which, revealingly, also shows acquaintance with *An. Ar.* in its account of Pythagoras' philosophy. Many independent lines of evidence point to the conclusion that, after the first innovative step made by Speusippus, arithmology disappeared from the historical scene for about two centuries, whereas number symbolism continued to reproduce old and accumulate ever newer confirmations of the power of significant numbers. *An. Ar.* can be accounted for only in the context of the decisive philosophical turn in the first century BC which gave rise to Neopythagoreanism and added to the pseudo-Pythagorean writings a heavy touch of Middle Platonic metaphysics.

4 The first century BC

Starting from Antiochus of Ascalon's (ca. 130–68 BC) revival of the teachings of Plato and the Early Academy, namely of Speusippus and Xenocrates, to which Aristotle was added as a true "early Platonist," we constantly hear about *symphonia* between fundamental doctrines of Plato and Aristotle (Karamanolis 2006). It is from the point of view of this *symphonia* that the alleged doctrines of the ancient Pythagoreans begin to be conceived. "The content of the pseudo-Pythagorean writings results from a blending of Platonist and Aristotelian doctrines which is typical of Platonism beginning in the first century BC" (Centrone 2014, 336–337). Indeed, the most conspicuous feature that *An. Ar.* shares with the other pseudo-Pythagorean works of this time is Middle Platonism very superficially disguised as ancient Pythagoreanism. In this sense arithmology is just an offshoot of the interest aroused in Pythagoras and the Pythagoreans, in the first place among Platonically inclined philosophers. They tried to satisfy this interest by means which became available precisely in that time. Among the principal sources they used were, firstly, the oral teaching of Plato as presented by Aristotle, Speusippus, Xenocrates, and other early Academics, and, secondly, Aristotle's critical description of the Pythagorean theories in the *Metaphysics* as well as in other treatises and exoteric works. Thirdly, for the arithmological line of the tradition, Speusippus' *On Pythagorean Numbers* was of special significance, for it is here that the foundations of arithmology were laid. All these sources, as we see, belong to the second half of the fourth century BC, but it was not until two and half centuries later, that due to a new approach to them, a complex picture of ancient Pythagoreanism and its legendary founder emerged.

This newly created Pythagoras came to possess a combination of distinctive features, a key part of which had existed previously, but not necessarily in connection with his name. 1) First and above all, Pythagoras is concerned with numbers, which complies with the substance of the Pythagorean theories presented by Aristotle, though the latter never related them to Pythagoras himself. 2) So understood, Pythagoras is regarded mainly as the predecessor and teacher of Plato, along with Socrates, but more essential for Plato's later metaphysics. This is again what we find in Aristotle and the early Peripatetics, but not, for that matter, in the early Academics. 3) The principal doctrine of this Pythagoreanism (as preliminarily Platonized by Aristotle) is identified as Plato's theory of the two opposite principles, the Monad and the Indefinite Dyad, which is regarded as having been anticipated by Pythagoras. This point plainly contradicts the position both of Aristotle and of the early Academics, for they never projected this Platonic theory onto Pythagoras or the Pythagoreans.[27] This is, then, a completely new feature. 4) This dualistic theory is subjected to monistic interpretation, so that either the Monad is conceived as producing the Indefinite Dyad, or the third, highest principle is set above the basic opposites, as, for example, in the accounts of the Pythagorean theories by Eudorus of Alexandria (fl. about 25 BC) and Moderatus of Gades (first century AD).[28] Nothing of this sort is attested in the earlier sources. In order to elucidate the background against which the emergence of the arithmological genre is to be understood, I shall comment on each these points, proceeding in reverse order.

The tendency to attribute to Pythagoras or the Pythagoreans the Platonic doctrine of the Monad and the Indefinite Dyad appears for the first time in the *Pythagorean Hypomnemata* (turn of the first century BC),[29] transmitted by the grammarian Alexander Polyhistor (worked in Rome after 82 – about 35). The Pythagorean theories of the *Hypomnemata* (D. L. 8.24–35) are fairly heterogeneous and eclectic and this concerns in the first place the doctrine of principles:

The principle of all things is the Monad. Arising from the Monad, the Indefinite Dyad serves as matter for the Monad, which is its cause (ἐκ δὲ τῆς μονάδος ἀόριστον δυάδα ὡς ἂν ὕλην τῇ μονάδι αἰτίῳ ὄντι ὑποστῆναι). From the Monad and the Indefinite Dyad arise numbers, from numbers points, from points lines, from lines plane figures, from plane figures solid figures, from solid figures sensible bodies, the elements of which are fire, air, earth, and water (8.25).

27 Burkert 1972, 62–65, 81–83; cf. Zhmud 2012, 421–432.

28 Eudorus: Simpl. *In Phys.* 181.7–30 = fr. 3–5 Mazzarelli; Moderatus: Simpl. *In Phys.* 230.34–231.24. See Doerrie / Baltes 1996, text: fr. 122.1–2, commentary: 473–485. See also Archytas, *De princ.* 19–20 Thesleff. Syrianus' commentary on the *Metaphysics* (166.3–8 Kroll) ascribes a similar triad – a highest principle above *peras* and *apeiria* – to Archaenetus (otherwise unknown), Philolaus and Bro(n)tinus (*De intell.* fr. 2 Thesleff), relying, therefore, on the pseudo-Pythagorean writings. See Merlan 1967, 84.

29 On the discussion of the dating, see Zhmud 2012, 423 n. 34.

As we see, the familiarly Platonic looking derivation of physical bodies from geometrical figures and numbers and ultimately from two highest principles has been revised here in the spirit of monism. This violated the original equality of the opposite *archai* by making the active Monad produce the Indefinite Dyad; the latter, respectively, became passive and material. Two basic tenets reveal the Stoic provenance of this way of thinking:[30] the Stoics maintained the difference a) between two *archai*, an active incorporeal principle (τὸ ποιοῦν) identified with reason (*nous*) and God, and a passive corporeal principle (τὸ πάσχον), identified with matter, and b) between ungenerated and indestructible *archai* and physical *stoicheia*. To be sure, Stoicism retained the fundamental dualism of its *archai*, in the sense that God never produces matter itself. In the realm of numbers and numerical principles, however, it seemed much easier for the Dyad to arise from the Monad, for this is exactly what happens in arithmetic. In an overview of the Pythagorean doctrines in Sextus Empiricus it is said that when the Monad is added to itself it produces the Indefinite Dyad (*Math*. 10.261). The Anonymus Photii (late first century BC) offers a still more resolutely monistic and mathematized version, where the Dyad is pushed far into the background (238a8–11).

This kind of Stoicized Platonism is manifested even more clearly in what the *Vetusta placita*, compiled in the school of Posidonius, passed off as Pythagoras' first principles: μονάς = τὸ ποιητικὸν αἴτιον καὶ εἰδικόν, ὅπερ ἐστὶ νοῦς ὁ θεός; ἀόριστος δυάς = τὸ παθητικόν τε καὶ ὑλικόν, ὅπερ ἐστὶν ὁ ὁρατὸς κόσμος.[31] In the later accounts some of the Stoic features recede to the margins, but they undoubtedly belong to the original setting of this Neopythagorean system. Its author remains unknown; to my knowledge, he has never been even tentatively identified. At any rate, the system must have been created by a single mind rather than simultaneously by several authors. E. Zeller put its appearance at the turn of the first century BC, and that dating remains the most plausible.[32] A Middle Platonic–Neopythagorean milieu, where Pythagoras was regarded as the predecessor of Plato's mathematically tinted metaphysics, and as the legendary sage whom Greek philosophy had to thank for all that was best in it, seems to provide the most natural context for this innovative doctrine. Thus, a decisive first step was made towards a new kind of Pythagoreanism which P. Merlan (1967, 91) aptly called "aggressive," for it laid claim to priority in the well known doctrines of Plato and the early Academics, Aristotle, and the Stoics. Reflecting changes of the philosophical climate, in the next two centuries this doctrine gained wide popularity, being attested in many pseudo- and Neopythagorean writings, as well as in biography and doxography.[33]

30 For further references to this subject see Zhmud 2012, 423 n. 35.

31 Aët. 1.3.8 = *Dox*. 281a6–12; cf. 1.7.1. In Aetius, for the sake of brevity, the idea that the Monad generates the Dyad is omitted, but it can easily be restored.

32 Zeller 1919 I, 464–467; III.2, 103–106. For further discussion, see Zhmud 2012, 423 n. 34.

33 Pseudo-Pythagoreans (quoted by page and line of Thesleff 1965 edition): Anonymus Alexandri (D. L. 8.25); Anonymus Photii (237.17–23, 238.8–11); Bro(n)tinus (*De intell*. fr. 2); Callicratides (fr. 1, 103.11); Pythagoras (*Hieros logos* in Doric prose, fr. 2, 164.24–26); Archytas (*De*

Now, in our sources this metaphysical system often appears accompanied by easily recognizable arithmological ideas. Though absent in the *Hypomnemata* (probably, because of their very concise exposition of principles) they are presented in three other important accounts of Pythagorean philosophy: Aetius (i. e. his source, the *Vetusta placita*), the Anonymus Photii, and Sextus Empiricus. Aetius (1.3.8) and the Anonymus Photii (238.1–3) state, for example, that the decad is the nature of number since all people count to ten and then turn back to the one; that the four is the decad δυνάμει and therefore is called the *tetractys*, and so on. Aetius (1.3.8) and Sextus Empiricus (*Adv. Math.* 4.2; 7.94) quote two verses of the famous Pythagorean oath, where the tetractys, the "source of everlasting nature," is attested for the first time; the Anonymus Photii also refers to the tetractys (238.1–3). According to Robbins' (1920, 310–315) convincing suggestion, the oath, clearly alluded to in Philo as well,[34] formed part of an introduction to the pseudo-Pythagorean arithmological treatise. The existence of *An. Ar.* is, therefore, presupposed in these sources.

This is one side of the coin. On the other side, all the arithmological writings starting with Philo's work *On Numbers*, the earliest and the most complete specimen of the genre (Staehle 1931, 1–11), contain conspicuous traces of the metaphysical system described above. Those that occur most frequently among them are the following:[35] the Monad by its nature is equal to God and reason (4a–c, h); it generates all the other numbers but is not generated in itself (5e); the Dyad "flows" from the Monad (8); the Dyad embodies the material principle (11b). This deep interpenetration of the doctrines leaves no doubt that arithmology as a genre and the Neopythagorean system of principles derive from the same philosophical milieu. They share a great deal in common, including their presuppositions and sources, on which we shall dwell later. By the early first century BC the system must have already been formed, for it is reflected in the *Pythagorean Hypomnemata* and the *Vetusta placita*.[36] From such a perspective, *An. Ar.* looks like an offshoot of the newly developed number metaphysics, with a more narrow focus on number speculations of various kinds – in the same way as the book of Speusippus, the *Urvater* of arithmology, arose against the background of Plato's number philosophy. In the late fourth century Aristotle's criticism, and the emergence of the new philosophical systems, Stoicism and Epicureanism, changed the atmosphere in the Academy and made number speculations obsolete. Middle Platonism and Neopythagoreanism successfully brought them back and made them an integral part of their philosophizing.

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princ. 19–20). Influenced by Neopythagoreanism: Eudorus (Simpl. *In Phys.*, 181.7–30). Neopythagoreans: Moderatus (ibid., 230.34–231.24); Numenius (fr. 52 Des Places). Doxography: Aët. 1.3.8 (= *Dox.* 281.6–12) and 1.7.18; Anonymus in Sextus Empiricus (*Adv. math.* 10.261–262).

34 See below, 341 n. 70.

35 Numbers in brackets refer to Staehle's 1931 collection of the parallels to Philo's arithmology.

36 Centrone 2014, 336–337 follows Zeller 1919, III.2, 113–114, in suggesting Alexandria as the most probable site of its emergence.

5 Plato Pythagoricus

One of the central presuppositions of the system described above consists in featuring Plato as the legitimate successor of Pythagoras and the Pythagoreans in a fully positive way, in defiance of what was told about this connection before. To be sure, an influential theory developed by Burkert (1972, 82) states that it was already Speusippus, Xenocrates, and Heraclides who equated "the doctrine of their master Plato, and therewith also their own philosophical positions, with the wisdom of Pythagoras." This theory implies that Speusippus and Xenocrates were the fathers of Neopythagoreanism, and they are so treated, for example, by J. Dillon.[37] However the evidence available to us does not support the thesis that the early Academics projected Plato's unwritten doctrine onto Pythagoras.[38] Indeed, Plato himself blurred over his dependence on the Pythagoreans, so why should the Platonists understate the originality of their teacher, who mentioned Pythagoras just once and even then only as an originator of the 'Pythagorean way of life' (*Resp.* 600a–b)? Revealingly, in the fourth century and later, Plato's dependence on the Pythagoreans (not yet Pythagoras himself!) is affirmed in a tradition that is either critical of him, in Aristotle and the Peripatetics, or openly hostile, in stories of his plagiarism from the Pythagoreans.

The idea of plagiarism evolved roughly in the following way.[39] The historian Theopompus, a student of Plato's chief rival Isocrates, in a special work against Plato, was apparently the first to accuse him of plagiarizing not the Pythagoreans – it is true – but Aristippus, Antisthenes, and Brison (*FGrHist* 115 F 259). This idea was taken up by Aristotle's student Aristoxenus, who asserted that Plato copied his *Republic* from Protagoras (fr. 67 Wehrli). Whether he accused Plato of copying from the Pythagoreans, we do not know, but in the succeeding generation this version was popularized by Neanthes and Timaeus of Tauromenium (D. L. 8.54–55). A slightly later version, that Plato had copied his *Timaeus* from Philolaus' book, has reached us via Timon of Phlius (fr. 54) and the biographer Hermippus (D. L. 8.85), whereas Satyrus replaced Philolaus' book with Pythagoras' *tripartitum*, mentioned above.[40] This made Plato entirely dependent on Pythagoras himself. Obviously, most of these stories come from the biographical tradition, which, beginning with its founder Aristoxenus (frs. 32, 62, 131), was very much disposed to inventing malicious anecdotes about Plato. Unsurprisingly, the attitude of the Early Academy was the direct opposite. The Seventh Letter attempts to prove, it seems, that Archytas (who never appears in the dialogues) was much weaker than Plato in philosophy and therefore could not have had any influence on him (Lloyd 1990). According to the Academic legend of the mid-fourth century, the famous problem of doubling the cube was solved by Archytas, Eudoxus, and

37 Dillon 1996, 38; Dillon 2003, 204.
38 For fuller discussion of the sources, see Zhmud 2012, 421–432.
39 See Brisson 1993; Dörrie / Baltes 1996, 473–485.
40 Above, 326; see Schorn 2004, 358–364 (F 10).

Menaechmus working under instructions from Plato and under his control.[41] A contemporary Academic source, preserved in Philodemus' *History of the Academy*, ascribes to Plato an even more significant role as architect of the mathematical sciences: "At this time *mathemata* were also greatly advanced, with Plato being the architect of this development; he set problems for the mathematicians, who in turn eagerly studied them."[42] The picture of Plato giving instructions to Archytas, the latter's student Eudoxus, and Eudoxus' student Menaechmus was further embellished in Eratosthenes' dialogue *Platonicus*. This, then, was the attitude of Plato and the Early Academy towards Pythagorean mathematics.

In the first century BC the situation radically changes, so much that Plato's intellectual indebtedness to Pythagoras was not only willingly recognized but became a cornerstone of later Platonism. Cicero, following a new biographical vulgate, several times reports the same narrative: Plato came to Italy and Sicily in order to meet the Pythagoreans and to appropriate their dogmata, of which Socrates had not even wanted to hear; Plato became acquainted with Archytas, Echecrates, and Timaeus of Locri, got access to Philolaus' book, learned all the Pythagorean teaching, first of all their *mathemata*, and made it more argumentative; out of love for Socrates, however, he ascribed this Pythagorean *sapientia* to his teacher.[43] Therefore, Plato becomes an acknowledged *diadochos* of Pythagoras and a student of Archytas, in which role he figures in Pythagoras' biography in the Anonymus Photii (237.5–7); Aristotle here turns into the next *diadochos*, which is logical from the perspective of the *symphonia* between him and Plato which had recently been asserted. This biographical pattern has undoubtedly been modified in order to adjust to the new interest in Plato's number metaphysics, because it could be accounted for only by his heavy debt to Pythagoreanism. The natural consequences of this new approach can be seen in the retrospective projection of Plato's (Stoically coloured) doctrine of principles, the Monad and the Indefinite Dyad, onto Pythagoras and, which is even more relevant for us, the appropriation of Speusippus' and other Early Academic arithmological schemes in the process of creating the Neopythagorean arithmology. But before coming to this issue it is important to recall the exceptional role of Aristotle in the appearance of the image of Plato Pythagoricus, because, historically speaking, this image was not in fact new in the first century BC.

6 Aristotle on Plato and the Pythagoreans

However sceptical an attitude one may have towards the story of Neleus of Scepsis' cellar as the *only* place where Aristotle's and Theophrastus' esoteric works were preserved, it is clear that in the third and second centuries BC they had fallen out of circulation. Even if some Hellenistic library did possess a copy of

41 See Zhmud 2006, 82–108, with a bibliography of the question.
42 Dorandi 1991, 126–127; Zhmud 2006, 87–89.
43 *Resp*. 1.15–16; *Tusc*. 1.39; *Fin*. 5.86–87. Dörrie / Baltes 1996, 250–256, 526–536.

what we know as Aristotle's *Metaphysics*, there is no evidence whatsoever that it was read and produced a philosophical reaction.[44] The growing awareness of Aristotle's importance during the first century BC was prompted, though not exclusively, by two editions of his principal esoteric treatises, first by Apellicon of Teos (ca. 100–90), and then by Andronicus of Rhodes (ca. 70–60), of which the latter subsequently became canonical. The rediscovered corpus of Aristotle's writings offered a philosophical portrait of Plato significantly different from that known from the dialogues. As distinct from Plato's tendency to obfuscate his debt to his predecessors, Aristotle regularly presented him, especially in the doxographical overview in *Metaphysics* A 3–7, as following the Pythagoreans in his *Prinzipienlehre*.[45] To Aristotle, Plato's unwritten doctrine of principles acquires its historical meaning only against the background of Pythagorean teaching and vice versa: the basic function of Pythagorean number doctrine lay in serving as the main source of Plato's late metaphysics. The pithy words attested for the first time in Aetius, Πλάτων δὲ καὶ ἐν τούτοις πυθαγορίζει, can easily be put into Aristotle's mouth, for he insistently pointed out the kinship between the doctrines of Plato and the Pythagoreans, while noting their *differentia specifica*. But sometimes, as for example in his report of Plato's famous lecture on the Good, he portrays them as being practically indistinguishable, thus taking a decisive step towards the metaphysical doctrine known to us from the first-century sources:

> Both Plato and the Pythagoreans assumed numbers to be the principles of the existing things, because they thought that that which is primary and incomposite is a first principle, and that planes are prior to bodies…, and on the same principle lines are prior to planes, and points (which mathematicians call *semeia* but they called units) to lines, being completely incomposite and having nothing prior to them; but units are numbers; therefore numbers are the first of existing things.[46]

It is easy to recognize that this Platonic and Early Academic derivation of points from units, which is to say numbers, and the further generation point – line – plane – body exactly corresponds to what the *Pythagorean Hypomnemata* and other philosophical and arithmological sources pass off as the doctrine of the Pythagoreans. If we add to this the first principles of numbers, the Monad and the Dyad, mentioned a bit later, the match becomes perfect.

Aristotle's students adopted from him a tendency to see Plato as the follower of the Pythagoreans. Thus the statement of Dicaearchus, that Plato in his teaching combined Pythagoras and Socrates (fr. 41), is a direct echo of the description of Plato in the *Metaphysics* (987a–b13).[47] Eudemus in his *Physics* (fr. 60) compares Archytas' idea, that the causes of motion are ἄνισον and ἀνώμαλον, with Plato's

44 Düring 1968, 192; Moraux 1973, 3–44; Gottschalk 1997, 1085; Sharples 2010, 24–30.
45 *Met.* 987a31. b10. b22, 990a30; see also 996a6, 1001a9, 1053b12.
46 Alex. *In Met.* 55.20–27 = *De bono*, test. and fr. 2 Ross. Cf. below, 337.
47 Cf. above 333 n. 43.

Prinzipienlehre, with his preference going to Archytas; he also praises the Pythagoreans and Plato for relating ἀόριστον to motion. Theophrastus in his *Metaphysics* (11a27–b10) lumps together Plato and the Pythagoreans by ascribing to them Plato's doctrine of ἕν and ἀόριστος δυάς. Together with Aristotle's *De bono* (fr. 2), this text constitutes the closest antecedent to what in Neopythagoreanism became a standard view.

Thus the first century BC was the moment at which all the relevant lines of influence intersected and supplemented each other, and a number of important developments took place that would eventually give rise to arithmology as a genre: the revival of a Platonism that included the theories of Speusippus and Xenocrates; the rediscovery of Aristotle, this time as a Platonist; the reappearance of Plato as the follower of the Pythagoreans, but now in a positive sense, and the corresponding transformation of Pythagoras into an author of Platonic and Early Academic number philosophy, which with some new features added became the metaphysical foundation of Neopythagorean arithmology – all this did not exist before the first century. This dooms to failure any attempt to connect arithmology directly with ancient Pythagoreanism. In order to find out whether or not an *indirect* connection is possible, we have to go back again to the fourth century, namely to the Early Academy.

7 Τέλειος ἀριθμός and the birth of the arithmological system

Number symbolism does not possess any intrinsic limit to significant numbers. Though they naturally tend to concentrate within the first decad, other numbers like 12, 13, 30, 40, and 50 could be equally important. The number ten in itself, although important in counting, does not play any noticeable role in traditional number symbolism. Unlike three, four, or seven, the number ten is not a symbol of a particular notion, thing, or group of things. Its symbolism is purely mathematical and its completeness, unlike the completeness of the three that stands for "all," consists of embracing "the entire nature of numbers." Beyond the world of numbers it does not seem to correspond to anything, so that the Pythagoreans, according to Aristotle, had to invent a new heavenly body for it! In this sense the birth of arithmology can be conceived as the process of limiting the traditional lore by a new conceptual framework imposed on it by an influential philosophical doctrine which attached great value to the number ten.

The doctrine in question is, of course, the unwritten doctrine of Plato, which comprises a theory of ten ideal numbers, or Forms-Numbers. Their generation serves as a model for the generation of all other numbers. When Aristotle refers to the theory of the ten archetypal numbers, he obviously has Plato in mind,[48] and in *Physics* 206b27–33 he directly names Plato (μέχρι γὰρ δεκάδος ποιεῖ τὸν ἀριθμόν). This is why the decad was counted as the perfect, or complete number. To be sure,

48 1073a17–22; 1084a12–b2: πειρῶνται δ᾽ ὡς τοῦ μέχρι τῆς δεκάδος τελείου ὄντος ἀριθμοῦ (a31); 1088b10–11. On Plato's teaching on decad, see e. g.: Dillon 1996, 4–5; Erler 2007, 427–428

in Plato's dialogues ten was not yet called a perfect number; τέλειος ἀριθμός refers in one case to the so-called nuptial number, and in the other to the great year.[49] This suggests, inter alia, that before Plato there was hardly any doctrine on the decad as τέλειος ἀριθμός. It appears for the first time in Speusippus' *On Pythagorean Numbers*, half of which was devoted to the marvellous properties of the decad (fr. 28). Speusippus rejected the theory of the Forms and replaced the ideal numbers with mathematical ones, and Xenocrates identified the ideal and the mathematical numbers, so that for them "*mathemata* have become a philosophy, although they say that *mathemata* should be studied for another reason" (Arist. *Met.* 992a31). A comparison of the basic features of arithmology, as reflected in Philo's work *On Numbers* and the abundant parallels to it in the later texts, with the theories of Speusippus and Xenocrates reveals how much this genre owes to them.

Whereas *An. Ar.* comprised an introduction and ten chapters devoted to the respective numbers, Speusippus' work does not yet exhibit this form, which is classical for arithmological literature. Speusippus' work consisted of two parts, the first of which dealt, according to the excerptor, with different kinds of numbers: linear, plane, solid, and so on, continuous and discontinuous proportions, and the five regular solids. In the second part appear other types of numbers, such as prime and composite, as well as multiple and epimoric ratios, and numerical progressions. At first sight, the subject looks more arithmetical than arithmological, but Speusippus' treatment of it was mathematical only to a very limited extent. He could assert, for example, that in an equilateral triangle in a certain sense there is one side and one angle! Saying that in the decad there are equal numbers of prime (1, 2, 3, 5, 7) and composite (2, 4, 6, 8, 10) numbers, he makes one a prime number, although in that case all the other numbers become composite (see Euc. 7, def. 12, 14). Anyway, most of these things go back to Pythagorean arithmetic, harmonics, and geometry (three regular solids were also constructed by them), and, if the title *On Pythagorean Numbers* is Speusippian (which is not certain: Tarán 1981, 262), it most probably referred to mathematical material which he used for his own paramathematical purposes.[50]

In the second part, better known to us thanks to a two-page quotation from it, Speusippus sets out his variant of the Academic doctrine of the decad, thus laying the foundations for the arithmological system. Its most conspicuous feature is that he focuses not on the correspondences between numbers and things, but on numbers and geometrical figures themselves and the interconnections between them. Such an emphasis is perfectly understandable insofar as for Speusippus numbers constitute the first layer of beings,[51] with magnitudes coming after

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49 *Resp.* 546b–d; *Tim.* 39d3–4. In mathematics τέλειος ἀριθμός is equal to the sum of all its divisors, e. g. 6 = 1 + 2 + 3, but this meaning is not attested before Euclid (7, def. 22; 9, 36).

50 "It is intelligible, then, that he should have called the 'linear', 'triangular' etc. numbers 'Pythagorean numbers'" (Tarán 1981, 263). On Speusippus' independence from the Pythagoreans see Tarán 1981, 109, 260, 269–276; Huffman 1993, 361.

51 Arist. *Met.* 1083a23 = Speus. fr. 34, 1075b37–1076a3 = fr. 30; 1080b11–16 = fr. 33.

them. He does not seem to be primarily motivated by traditional number symbolism: the numbers three, seven, or nine do not interest him as such; instead he is fixated on four and ten, since they are the pillars of Platonic number metaphysics. Speusippus' interest was primarily philosophical and this is what gave arithmology a completely new dimension. In fact, his deliberate focus on mathematicals was too radical and refined to be directly followed in a popular philosophical genre. The author of *An. Ar.* had to take a considerable step back by returning again to traditional number symbolism and applying it to the conceptual framework created by Speusippus. What we observe in the later arithmological texts is, as it were, Speusippus "lite:" they are not so heavily metaphysically loaded and contain much entertaining material on the parts of the human body, seven- and nine-month babies, and so on.

According to the Academic doctrine, ontological priority resides with that which can exist without another. Bodies are less substance than planes, planes than lines, lines than points, and points than units,[52] since "a unit is substance without position, while a point is substance with position," which is to say that the latter contains an additional property.[53] Thus, numbers are by nature first. Respectively, the line is derived from the point (a variant: is produced by a moving point, *De an*. 409a4–7), the plane from the line, and the body from the plane, and this derivation sequence is closely connected to the first four numbers, for Speusippus, for example, associated the point with one, the line with two, the plane with three, and the pyramid with four. Schemes of generation of magnitudes are attested for Speusippus and Xenocrates,[54] and Aristotle attributed to Plato the derivation of line, plane, and solid "after numbers" or even from numbers.[55] In his tract Speusippus tirelessly connects the number four with the decad, being very enthusiastic about the transformation of the tetrad into the decad: 1 + 2 + 3 + 4 = 10. The number ten contains all kinds of number, he asserts,

> including the linear, plane, and solid numbers. For 1 is a point, 2 is a line, 3 is a triangle, and 4 is a pyramid; all these are elements and principles of the figures like them. In these numbers is seen the first of progressions… and they have 10 for their sum. The primary elements in plane and solid figures are point, line, triangle, pyramid, they contain the number ten and are limited by it (fr. 28).

Arithmology echoes this scheme by regularly equating one with the point, two with the line, three with the triangle, and four with the pyramid.[56] The dyad is

52 Arist. *Met*. 1002a4–8, 1019a1–4; 1017b6–21; *De bono* fr. 2 (above, 334 n. 46).
53 Arist. *APo* 87a35–37 See also *Met*. 982a26–28 and above, 334. A point as a monad having position is an Academic formula (Burkert 1972, 67).
54 According to Speusippus, a point is the *arche* of line (Tarán 1981, 268). Xenocrates fr. 117 Isnardi Parente.
55 *De an*. 404b19–24. See also Arist. *Met*. 1090b21–24 = Xenocr. fr. 38 Isnardi Parente.
56 6a–b (one), 14a–c (two), 19a–e (three), 26a–d (four). The numbers here refer to Staehle 1931.

generated by the "flow" (ῥύσις) of the monad, the line by the "flow" of the point, and the plane by the "flow" of the line.[57] The tetrad is the "origin" and "source" of the decad (47a–b). Seven within the decad is neither a factor nor a product.[58] The decad is most perfect, it encloses all types of numbers and numerical relations; all people count to ten and then turn back.[59] This common stock of arithmology goes back to Speusippus. That the addition of odd numbers produces a square number, while the addition of even numbers produces an oblong number (13a), is once alluded to in Aristotle with reference to the Pythagoreans (*Phys.* 203a3–16), but his notice is very unclear; it is more likely that the source of this was Speusippus, who treated plane numbers in his book.

According to Aristotle, the matching of various types of cognitive activity to the first four numbers (νοῦς 1, ἐπιστήμη 2, δόξα 3, αἴσθησις 4) is derived from Plato, who put forward these types themselves.[60] In the *Timaeus* (47e) the Demiurge is identified with *nous*. Xenocrates, following Plato, identified *nous* with τὸ ἕν (fr. 213) and with God, and Speusippus declared God to be *nous* (fr. 58). Arithmology invariably associates the monad with God and mind (4a–c, h), whereas other correspondences are more fluid. The doctrine that the dyad is the first female number and the triad the first male number also seems to originate with Xenocrates, who assigned such predicates as ἄρρεν–θῆλυ and περιττὸν–(ἄρτιον) to his first principles Μονάς and Δύας.[61] Aristotle agreed with the Pythagoreans that three is τέλειος ἀριθμός, for it signifies totality (see above, 324); he could have been the source of this idea in arithmology.

8 Pythagorean roots of arithmology?

If the conceptual foundations of the arithmological system were laid down by Plato and his students Speusippus and Xenocrates, what then was the historical role of the Pythagoreans in the formation of the intellectual tradition which is so firmly and universally connected with them? Pythagorean arithmology stands or falls with Aristotle's account of Pythagorean number philosophy, for he was the only one who ascribed to the (unnamed and unknown) Pythagoreans such notions as the significance of the decad, the likening of types of cognitive activity to numbers, and so on. Other classical sources are silent on this. What is more important is that, in the authentic fragments of the individual Pythagoreans and in the reliable evidence on them, arithmology is not to be found, as distinct from the traditional number symbolism.[62] One of the early responses to Solon's elegy on seven-year periods has come down from Alcmaeon of Croton, who stated that

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57 8a, 14a–c, cf. Speus. fr. 52.
58 43a–k, cf. Speus. fr. 28, l. 30.
59 86a–c, 87a–b, 88–89a–k, 90a–b, 92.
60 *De an.* 404b19–24. On types of cognition in Plato: *Phaed*. 96b, *Parm*. 142a, 151e, 164a, *Tim*. 37b–c, *Phil*. 21b; in Aristotle: *APo* 88b34–89a2, 100b4–17; *De an*. 428a3; *Met*. 1074b34–36.
61 Aët. 1.7.30 = fr. 213; Dörrie / Baltes 1996, 192–194; Dillon 2003, 99–107.
62 The fragments on the decad of Philolaus (A 11–13, B 11) and Archytas (B 5) are spurious.

young men achieve sexual maturity at the age of twice seven (24 A 15). To this division of life into periods of seven years, Presocratic philosophy and Hippocratic medicine added analogous notions regarding the development of the foetus, divided into weeks and months. A similar combining of the embryological calendar with the division of life into periods of seven is found in Hippon, the Pythagorean natural philosopher of the mid-fifth century. In an attempt to take into consideration data derived from experience, in his calculations, in addition to the number seven, he makes use of the still more significant number three:

> <…> After the seventh month, our teeth begin to emerge and then they fall out in the seventh year; <…> But this maturity which begins in the seventh month is prolonged to the tenth, because the same natural law applies to everything, so that three months or years are added to the original seven months or years to bring things to completion. So the child's teeth are formed in the seventh month but not completed until the tenth; the first teeth fall out in the seventh year, the last in the tenth; most have reached puberty after fourteen years, but everyone has by seventeen (38 A 16, tr. H. N. Parker).[63]

Obviously, Hippon or any other Pythagorean could have had a preference for seven or three, but such preferences are not in themselves Pythagorean: Aristotle also had a predilection for both these numbers. Usually critical of the Pythagoreans, he concurred with them on the triad (*Phys*. 268a10–20) and insisted that the rainbow necessarily has only three colours (*Mete*. 374b28–375a7). Similarly, the number of colours, tastes, and vowels necessarily equals seven (*De sensu* 442a19–25, 446a19). Theophrastus adds odours to colours and tastes, calling the number seven καιριώτατος καὶ φυσικώτατος (*CP* VI,4,1–2). The Pythagoreans also connected καιρός with the number seven, making use of the same traditional notions as Aristotle and Theophrastus. In the same way they connected justice with the number four, because justice "returns like for like." In such and similar examples which, in fact, are not as numerous as is usually believed,[64] we do not find specific features of arithmology, as described above. Pythagorean number symbolism has a pre-philosophical origin and mainly coincides with non-Pythagorean number symbolism.[65] Where numbers are conceived as the members of the series limited by the ten, we can detect the influence of the Academy.

Aristotle, however, regarded the Pythagoreans as the philosophical predecessors of Plato's unwritten doctrine (Zhmud 2012, 415–452). It seems only natural, then, that he was the first to ascribe to them directly the theory of ten as a perfect number:

63 What is important here, is the sum of seven and three: $7 + 3 = 10$, $7 + 7 + 3 = 17$, etc.

64 *Met*. 985b29–30, 990a23, 1078b22–23; *EN* 1132b23; *MM* 1182a11; fr. 13 Ross.

65 Pherecydes (7 B 1), Ion of Chios (36 B 1), and Hippodamus (39 A 1) attached special significance to the number three, Empedocles to four and seven (31 A 75, 83, B 153a).

> Since the number ten is considered to be τέλειος and to comprise the whole nature of numbers, they also assert that the bodies which revolve in the heavens are ten; and there being only nine that are visible, they make the counter-earth the tenth.[66]

This is, of course, only his interpretation of Philolaus' astronomical system, for how plausible is it that Philolaus would have devised an invisible planet solely for the sake of a round figure, and that he directly said so? Arithmology does not invent things, but fits them into numbers or derives numbers from things available, of which there are always sufficient to produce the desired combination. Elsewhere Aristotle gives another, astronomical explanation of Philolaus' motives for introducing the counter-earth (58 B 36), which is much more persuasive than an arithmological explanation (Zhmud 2012, 406–407). Further, Philolaus introduced *two* invisible heavenly bodies: Hestia, or Central Fire, and the counter-earth, which revolved with the earth around Hestia. Had he wished to bring the number of heavenly bodies to ten, he could have stopped with Hestia, which was the tenth. The counter-earth could only appear in his system after Hestia, hence being the eleventh heavenly body! Certainly Aristotle speaks of ten *rotating* bodies, leaving the stationary Hestia out of the ten. But if Philolaus had wished to count Hestia too, the fact that it was motionless would hardly have stopped him.

If the number ten in the eyes of the Pythagoreans had such magical power that for its sake Philolaus invented a new planet, this belief should have left numerous traces, similar to those left by the numbers three and seven. In fact, the only other example of this account is the famous table of the ten pairs of opposites that Aristotle ascribes to a separate group of Pythagoreans (*Met*. 986a22–b8). Most experts agree now that it contains both Pythagorean and Academic material (Burkert 1972, 51), it is only the proportions which are disputed. True, the table begins with the pair limit-unlimited, known from Philolaus, but does this guarantee its Pythagorean origin as a whole? Such pairs as warm and cold, dry and wet, sweet and bitter, typical of the Pythagoreans and the Presocratics in general, are absent from the table. The combination of even and odd with left and right first appears in Plato's *Laws* (717a–b). According to Aristotle, the pairs at rest and moving, and good and bad, are typically Platonic (*Met*. 1084a35), being derived from his ἀρχαί, the One and the Indefinite Dyad. One and plurality are not only a Platonic principle; they constitute the cornerstone of Speusippus' philosophy. The male-female pair was significant to Xenocrates, who linked it to another pair, even-odd (fr. 213). It is known that Speusippus and Xenocrates had a series of op-

66 *Met*. 986a8–12. It is worth noting that Aristotle does not speak of the Pythagorean *origin* of this doctrine; rather, he refers to an already existent theory which is supported also by the Pythagoreans.

posites similar to those of the Pythagoreans.[67] Aristotle himself evidently thought in terms of a universal table of opposites, of which the "Pythagorean table" was a particular instance. Sometimes he mentions it as if it were Academic.[68] Thus, however much the table ultimately derives from the Pythagorean tradition in its detail, in its final form of the *ten pairs of distinct kindred opposites*, it was created by somebody very well versed in the teaching of Plato and the Platonists.

The second pillar of arithmology is the tetrad, the "source" of the decad. In the Pythagorean tradition it is even less traceable than the decad, if we discard the likening of justice to reciprocity and thus to the number four. Revealingly, Aristotle mentions the tetrad only when discussing the generation of numbers and geometrical figures by Plato and the Platonists,[69] and never relates it to the Pythagoreans. Obviously, he knew nothing of the famous tetractys which in the modern scholarship figures as a "kernel of Pythagorean wisdom" (Burkert 1972, 72). Τετρακτύς is a special term for a group of the first four numbers which make ten (later, other kinds of tetractys were devised). Since the numbers of the tetractys express the ratios of the basic concords,[70] it was regarded as being intimately related to music; one of the Pythagorean "symbols," quoted by Iamblichus, says: "What is the oracle at Delphi? The tetractys, which is the harmony in which the Sirens sing" (*VP* 82). The tetractys may appear thoroughly archaic, but is in fact a Neopythagorean edifice. The ancient Pythagoreans did indeed assign special significance to the numbers that expressed concords, but in harmonics what interested them was not numbers as such, but their ratios, λόγοι. The fact that the ratios of the basic concords consist of the first four numbers, which add up to ten, is more likely to please lovers of arithmology, such as Speusippus, than a mathematician. The number ten plays no part in harmonics and, as I have tried to show, bears no relation to ancient Pythagoreanism.

The word τετρακτύς appears for the first time in the Pythagorean oath, which was quoted almost simultaneously by *An. Ar.* and the *Vetusta placita* (above, 331). In the same first century BC the τετρακτύς was mentioned by the Anonymus Photii (439a8) and alluded to in Philo.[71] The Pythagorean oath is a typical specimen of pseudo-Pythagorica:

> Οὔ, μὰ τὸν ἁμετέρᾳ κεφαλᾷ παραδόντα τετρακτύν
> παγὰν ἀενάου φύσεως ῥίζωμά τ᾽ ἔχουσαν (Aët. 1.3.8).

67 Speusippus: Arist. *Met.* 1085b5, 1087b4, b25; 1092a35. For Xenocrates one could reconstruct the following table of opposites: μονὰς–δυάς, ἄρρεν–θῆλυ, Ζεὺς–μήτηρ θεῶν, περιττὸν–ἄρτιον, νοῦς–ψυχή (fr. 213).

68 See for example: *Phys.* 189a1–5, 201b21–27; *Met.* 1004b27–35, 1093b11–14.

69 *Met.* 1081a23, b15–22; 1082a12–34, 1084a23; 1090b23.

70 2:1 the octave, 3:2 the fourth, 4:3 the fifth.

71 He sets forth in detail the same doctrine of the τέλειος τετράς as the decad *in potentia*, which Aetius attributes to Pythagoras: *De opif.* 47–53, 97–98; *De plant.* 123–125; *De vita Mosi* 2.115.

> No, I swear by him who gave the tetractys to our head,
> which has the source and root of everlasting nature.

Its spuriousness is clear from the pseudo-Doric dialect (φύσεως is an Attic form), and the verse form, which is not attested in authentic oaths, and the fact that Pythagoras is not named in it (according to Nicomachus, the Pythagoreans did not call Pythagoras by his name.[72] It is significant that before the mid-first century BC the expression φύσις ἀέναος is used only by Posidonius.[73] Xenocrates designated the second of his two principles ἀέναος (fr. 101), "ever-flowing," "everlasting," but one should not identify a reference to the Pythagorean oath here.[74] Ἀέναος is abundantly attested before Xenocrates, both in poetry and prose, in Plato amongst others,[75] and to connect it with the oath first attested in the mid-first century BC is quite pointless.

The only evidence that *could* save the historical authenticity of the τετρακτύς is the Pythagorean "symbol" which mentions the tetractys as the harmony of the Sirens. The tradition of the Pythagorean "symbols", to which Iamblichus attached the word (popular among modern scholars) *akousmata*, goes back to the archaic period and even earlier.[76] Some proportion of the "symbols" known in Antiquity did actually exist in the sixth–fifth centuries BC, but the problem with our symbol is that it is found *only* in Iamblichus and in no other ancient writer. Although the collection of symbols in Iamblichus' *De vita Pythagorica* 82–86 as a whole goes back to Aristotle's book *On the Pythagoreans*, it is clear that Iamblichus did not use Aristotle himself but an intermediate source, in which the early symbols may have been diluted by later ones. Now, it is not difficult to find out that the harmony of the Sirens (without the tetractys) figures twice in Plato's *Republic* (and *nowhere* earlier), in the passage in which the famous heavenly harmony is described.[77] Thus the symbol adduced by Iamblichus is not the "higher wisdom" of the ancient Pythagoreans, but a combination of Plato's harmony of the Sirens with the late Hellenistic pseudo-Pythagorean tetractys. The tetractys, for its part, arose from the tetrad extolled by Speusippus in his work *On Pythagorean Numbers*.

Presenting the Pythagoreans in the *Metaphysics* A, Aristotle mentions three concepts in which they saw "resemblances" with the numbers: ψυχὴ καὶ νοῦς, καιρός and δικαιοσύνη, but he immediately indicates that the list is open-ended (985b26–31). In Book M, however, he specifies that the Pythagoreans explained

72 Iamb. *VP* 88. The legendary phrase αὐτὸς ἔφα (ἔφα is Doric) that occurs first in Cicero (*ND* 1.10), belongs to the same pseudo-Pythagorean milieu.

73 Fr. 239 E–K. The publishers of his fragments see in this a reference to the Pythagorean oath, but the reverse influence seems more easily arguable on chronological grounds.

74 As Burkert 1972, 72 and Dillon 1996, 100.

75 See *LSJ*, s. v. ἀέναος; Crit. 88 B 18.1–2 DK; Plat. *Leg.* 996e2 (ἀέναος οὐσία).

76 For a full discussion of the symbols see Zhmud 2012, 192–206.

77 ἐπὶ δὲ τῶν κύκλων αὐτοῦ ἄνωθεν ἐφ᾽ ἑκάστου βεβηκέναι Σειρῆνα συμπεριφερομένην, φωνὴν μίαν ἱεῖσαν, ἕνα τόνον· ἐκ πασῶν δὲ ὀκτὼ οὐσῶν μίαν ἁρμονίαν συμφωνεῖν (617b4–7); πρὸς τὴν Σειρήνων ἁρμονίαν (617c4).

only a few things by means of numbers, such as καιρός, or justice, or marriage (1078b21–23). Justice and καιρός occur several times elsewhere,[78] marriage and ψυχὴ καὶ νοῦς only once; what numbers are attached to them, is not said. If we add to them the number three as the symbol of an "all" (*Phys.* 268a10–20), it will exhaust the list of the Pythagorean likenings of concepts to numbers which appear in the treatises of Aristotle and which he erroneously understood as philosophical definitions explaining the essence of the things. Mathematics is present here only insofar as two added to itself makes four, and the (Academic) decad does not figure in this context, for it was not attached to any concept. Three, four, and seven belong to the classical repertoire of number symbolism, so, if among the ancient Pythagoreans there were some people attached to these numbers, they would not appear much more superstitious than Aristotle himself. There is, however, one source which not only significantly enriches our knowledge of Pythagorean number symbolism, but in fact transforms it into an arithmological system. This is Alexander of Aphrodisias' commentary on Aristotle's *Met.* 985b26, where W. Ross, following P. Wilpert, identified an extensive quotation from Aristotle's work *Against the Pythagoreans* (38.8–41.15 Hayduck = fr. 13 Ross). Alexander presents the whole series of numbers from one to ten accompanied by explanations very similar to or identical with those of the arithmological texts.[79] Commenting on the passage where justice and καιρός appear, he started from the four and seven, but the original order is easy to restore.

One is νοῦς and οὐσία, "because one was unchanging (μόνιμον), alike everywhere, and a ruling principle, <…> but they also applied these names to substance, because it is primary." Two is δόξα, "because it can move in two directions; they also called it movement and *epithesis*;" two is also the first even number and female. Three is the first odd number and male. Four is justice and the first square number; but others declared justice to be nine, the first square of an odd number. Five is marriage, because it is the first number generated from two, which is male, and three, which is female. Seven is καιρός, since birth, the emergence of teeth, puberty, and so on are related to the number seven. Further, since the sun αἴτιος εἶναι τῶν καιρῶν, it is situated in the same place as the number seven, for, of the ten bodies which revolved around Hestia, the sun occupied seventh place. Seven is also Athena, the motherless maiden, because it alone among the numbers of the decad neither generates any number nor is generated from any. The moon occupies the eighth place, the earth the ninth, and the counter-earth the tenth.

Thus, the numbers three and seven are attested in the doxography on the historical Pythagoreans; the numbers three, four, and seven, and two more unidentified numbers appear in Aristotle's treatises; and the whole series from one to ten (except for six), with detailed explanations, is presented in an excerpt from

78 See above, 339 n. 63.

79 Asclepius' commentary on *Met*. 985b26 contains more or less the same material at slightly less extent (36.1–34.4 Hayduck).

his lost work. Which line of the tradition is more reliable, and are they mutually compatible? There are, I believe, many serious reasons to doubt that Alexander's excerpt represents a) Aristotle's account b) of Pythagorean views. If it derives from Aristotle, it contains, besides the Pythagorean material, many Academic notions, unattested in the independent Pythagorean tradition. Identification of *nous* with the number one is attested for Plato and Xenocrates.[80] Οὐσία is a typically Platonic, and later Peripatetic term: Plato contrasted οὐσία, immutable essence, to becoming and motion (*Tim.* 29c); in the *Cratylus* (411c5), μόνιμον is used in this same sense; Eudemus (fr. 60) reports that Plato identified κίνησις with "great-and-small," that is, with the Indefinite Dyad; thus, the entire contrast between the "unchanging" monad and "moving" dyad is Platonic.[81] Even if the "Pythagorean" definitions do not fully coincide with those of Plato (for him, opinion was three, not two), it is clear that we are dealing with an Academic type of arithmology. Sexual differentiation between even and odd numbers is attested in Xenocrates (above, 338); it seems unlikely that it goes back to an ancient tradition. At least, we have no evidence of this. The odd-even and male-female pairs, however, appear in the table of opposites, whose Academic provenance is not in doubt (see above, 340). Seven as Athena goes back to Speusippus, who claimed that seven was neither a quotient nor a divisor (fr. 28, l. 30). The very idea that numbers can be generated, so insistently repeated by Alexander, is typically Platonic.

Now, let us imagine for the sake of argument that some fourth-century Pythagoreans unknown to us did set forth such an oral doctrine before Plato and the Academy. Then it would have been available only to Aristotle (for nobody else testifies to it) and would have disappeared after him, leaving no traces in the classical and Hellenistic tradition except for the Early Academy. Again, this doctrine would have influenced the Academy in such a way that its distinctively Pythagorean features remained concealed – for Plato, Speusippus, and Xenocrates never say that justice is four and καιρός is seven – whereas all its "proto-Platonic" features became maturely Platonic. If such a case is hard to imagine, it is still possible to argue that Aristotle may have mistakenly ascribed the Platonic notions to the Pythagoreans (see above, 334). It is more problematic to maintain that the text, the kinship of which with the arithmological genre is more manifest than that of Speusippus' treatise, was written in the fourth century BC. Indeed, unlike Speusippus' work, Alexander's commentary displays all the typical features of an arithmological work. It is organized as a systematic commentary on the numbers from one to ten, not as scattered remarks on some significant numbers. It combines traditional number symbolism with ontology (substance, rest, movement, and so on) and mathematical arithmology: odd and even numbers, squares of them, ungenerated numbers, etc. It includes material on the number seven taken

80 See above, 338. Among the Pythagoreans νοῦς καὶ ψυχή appears only in Ecphantus of Syracuse, who makes it the force which constantly moves the whole cosmos (51 A 1).

81 Cf. a late ps.-Archytean passage: ἐπιστατὰ μὲν τὰ ἀκίνητα, δοξαστὰ δὲ τὰ κινεόμενα (36.19).

ultimately from Solon (see above, 324). Seven here occupies the most prominent place, as in all arithmological texts, and three different interpretations are given to it: naturalistic, as for example in Hippon, arithmological, as in Speusippus, and cosmological, based on Philolaus' system. Such a combination of the different sections of reality is distinctive of arithmological texts. Consequently, this Aristotelian fragment becomes an effective alternative to the origin of arithmology as described above, for it contains basically everything that arithmology is about and thus makes unnecessary the entire historical evolution of the genre. Another alternative would be to consider to what extent exactly this fragment is indeed Aristotelian.[82]

Bibliography

Barker, A. 2016, "Pythagoreans and Medical Authors on Periods of Human Gestations" (in this volume).

Brisson, L. 1993, "Les accusations de plagiat lancés contre Platon," in: M. Dixsaut (ed.), *Contre Platon*, vol. 1. *Le platonisme dévoilé*, Paris, 339–356.

Burkert, W. 1972, *Lore and Science in Ancient Pythagoreanism*, Cambridge Mass.

Centrone, B. 2014, "The Pseudo-Pythagorean Writings," in: C. A. Huffman (ed.). *The History of Pythagoreanism*, Cambridge, 315–340.

Collins, A. Y. 1984, "Numerical Symbolism in Jewish and Early Christian Apocalyptic Literature," *Aufstieg und Niedergang der Römischen Welt* II.21, Berlin / New York, 1221–1287.

Dawson, W. R. 1927, "The Number 'Seven' in Egyptian Texts," *Aegyptus* 7, 97–107.

Delatte, A. 1915, *Études sur la littérature pythagoricienne*, Paris.

Dillon, J. 1996[2], *The Middle Platonists*, London.

Dillon, J. 2003, *The Heirs of Plato*, Oxford.

Dorandi, T. 1991, *Platone e l'Academia (PHerc. 1021e 164) / Filodemo*, Napoli.

Dörrie, H. / Baltes, M. 1996, *Der Platonismus in der Antike*, Bd. 4, Stuttgart.

Dudley, U. 1997, *Numerology: Or What Pythagoras Wrought*, Cambridge.

Düring, I. 1968, s. v. Aristoteles, *RE Suppl*. XI, 159–336.

Edelstein, L. / Kidd, I. G. 1972–1988, *Posidonius, Fragments*, 3 vols., Cambridge.

Erler, M. 2007, *Platon*, in: *Grundriss der Geschichte der Philosophie. Die Philosophie der Antike*, vol. 2.2, Bern.

Gottschalk, H. B. 1987, "Aristotelian Philosophy in the Roman World," *Aufstieg und Niedergang der Römischen Welt* II.36.2, Berlin / New York, 1079–1174.

Holladay, C. R. (ed.) 1995, *Fragments from Hellenistic–Jewish Authors*. III. *Aristobulus*, Atlanta.

Huffman, C. A. 1993, *Philolaus of Croton. Pythagorean and Presocratic*, Cambridge.

Kalvesmaki, J. 2013, *The Theology of Arithmetic. Number Symbolism in Platonism and Early Christianity*, Cambridge Mass.

Karamanolis, G. E. 2006, *Plato and Aristotle in Agreement? Platonists on Aristotle from Antiochus to Porphyry*, Oxford.

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Lease, E. B. 1919, "The Number Three, Mysterious, Mystic, Magic," *Classical Philology* 14, 56–73.
Mansfeld, J. 1971, *The Pseudo–Hippocratic Tract* ΠΕΡΙ 'ΕΒΔΟΜΑΔΩΝ *Ch. 1–11 and Greek Philosophy*, Assen.
Mehrlein, R. 1959, "Drei," *Reallexikon für Antike und Christentum* 4, 269–310.
Merlan, Ph. 1967, "The Pythagoreans," in: A. H. Armstrong (ed.), *The Cambridge History of Later Greek and Early Medieval Philosophy*, Cambridge, 84–106.
Miller, G. A. 1956, "The Magical Number Seven, Plus or Minus Two: Some Limits on Our Capacity for Processing Information," *Psychological Review* 63, 81–97.
Moraux, P. 1984, *Der Aristotelismus bei den Griechen*, Bd. 1, Berlin.
Palmer, R. 1970, *The Archaic Community of the Romans*, Cambridge.
Parker, H. N. 1999, "Greek Embryological Calendars and a Fragment from the Lost Work of Damastes," *Classical Quarterly* 49, 515–534.
Reinhold, G. (ed.) 2008, *Die Zahl Sieben im Alten Orient*, Frankfurt a. M.
Robbins, F. E. 1920, "Posidonius and the Sources of Pythagorean Arithmology," *Classical Philology* 15, 309–322.
Robbins, F. E. 1921, "The Tradition of Greek Arithmology," *Classical Philology* 16, 97–123.
Robbins, F. E. 1926, *Nicomachus of Gerasa. Introduction to Arithmetic*, trans. into English by M. L. D'Ooge, with studies in Greek arithmetic by F. E. Robbins, New York.
Rohde, E. 1925, *Psyche: The Cult of Souls and the Belief in Immortality among the Greeks*, London.
Roscher, W. F. 1904, *Die Sieben- und Neunzahl in Kultus und Mythus der Griechen*, Leipzig.
Roscher, W. F. 1906, "Die Hebdomadenlehre der griechischen Philosophen und Ärzte," *Abhandlungen der Königlich Sächsischen Gesellschaft der Wissenschaften* 24.6 (1906).
Roscher, W. F. 1913, *Die hippokratische Schrift von der Siebenzahl in ihrer vierfachen Überlieferung*, Padeborn.
Runia, D. T. 2001, *Philo of Alexandria. On the Creation of the Cosmos According to Moses*, Leiden.
Schmekel, A. 1892, *Die Philosophie der mittleren Stoa*, Berlin.
Schorn, S. 2004, *Satyros aus Kallatis. Sammlung der Fragmente mit Kommentar*, Basel.
Sharples, R. W. 2010, *Peripatetic Philosophy, 200 BC to AD 200: An Introduction and Collection of Sources in Translation*, Cambridge.
Staehle, K. 1931, *Die Zahlenmystik bei Philon von Alexandria*, Leipzig.
Tarán, L. 1981, *Speusippus of Athens*, Leiden.
Thesleff, H. (ed.) 1965, *The Pythagorean Texts of the Hellenistic Period*, Åbo.
Usener, H. 1903, "Dreiheit," *Rheinisches Museum* 58, 1–55; 161–208, 320–362.
Walter, N. 1964, *Der Thoraausleger Aristobulos*, Berlin.
Waterfield, R. (trans.) 1988, *The Theology of Arithmetic. On the Mystical, Mathematical and Cosmological Symbolism of the First Ten Numbers*, Michigan.
Zeller, E. 1919[6], *Die Philosophie der Griechen in ihrer geschichtlichen Entwicklung*, 3 vols., Leipzig.
Zhmud, L. 2006, *The Origin of the History of Science in Classical Antiquity*, Berlin.
Zhmud, L. 2012, *Pythagoras and the Early Pythagoreans*, Oxford.
Zvi, G. 1988, "The Magical Number Seven," in: *Occident and Orient. A Tribute to the Memory of A. Scheiber*, Budapest / Leiden, 171–178.